| Sl. No. | Topics in JNTU syllabus | Modules and Sub modules | Lecture | Suggested books | Remarks |
|---|---|---|---|---|---|
| | | **UNIT-V** | | | |
| 5 | **Complex power series:** | Radius of convergence | L34 | T1-Ch5, R1-Ch5 | |
| | | Expansion in Taylor’s series | L35,36 | T1-Ch5, R1-Ch5 | |
| | | Maclaurin’s series and Laurent series | L37 | T1-Ch5, R1-Ch5 | |
| | | Singular point | L38 | T1-Ch5, R1-Ch5 | |
| | | Isolated singular point | L39 | T1-Ch5, R1-Ch5 | |
| | | Pole of order m | L40 | T1-Ch5, R1-Ch5 | |
| | | Essential singularity | L41 | T1-Ch5, R1-Ch5 | |
| | | (Distinction between the real analyticity and complex analyticity) | L42 | T1-Ch5, R1-Ch5 | |
| | | **UNIT-VI** | | | |
| 6 | **Contour Integration** | Residue | L43 | T1-Ch6, R1-Ch6 | |
| | | Evaluation of residue by formula and by Laurent series | L44 | T1-Ch6, R1-Ch6 | |
| | | Residue theorem | L45 | T1-Ch6, R1-Ch6 | |
| | | Evaluation of integrals of the type | L46 | T1-Ch6, R1-Ch6 | |
| | | Improper real integrals $\int_{-\infty}^{\infty} f(x)dx$ | L47 | T1-Ch6, R1-Ch6 | |
| | | $\int_{c}^{c+2®} f(\cos¯,\sin¯)d¯$ | L48 | T1-Ch6, R1-Ch6 | |
| | | $\int_{-\infty}^{\infty} e^{imx} f(x)dx$ | L49 | T1-Ch6, R1-Ch6 | |
| | | Integrals by indentation | L50 | T1-Ch6, R1-Ch6 | |
| | | **UNIT-VII** | | | |
| 7 | **Conformal mapping** | Transformation by $e^z$, lmz, $z^2$, $z^n$ (n positive integer) | L51 | T1-Ch7, R1-Ch7 | |
| | | Sin z, cos z, z + a/z | L52 | T1-Ch7, R1-Ch7 | |
| | | Translation | L53 | T1-Ch7, R1-Ch7 | |
| | | rotation | L54 | T1-Ch7, R1-Ch7 | |
| | | inversion and bilinear transformation | L55 | T1-Ch7, R1-Ch7 | |
| | | fixed point | L56 | T1-Ch7, R1-Ch7 | |
| | | cross ratio properties | L57 | T1-Ch7, R1-Ch7 | |
| | | invariance of circles and cross ratio | L58 | T1-Ch7, R1-Ch7 | |
| | | determination of bilinear transformation mapping 3 given points | L59 | T1-Ch7, R1-Ch7 | |

**7.6.11 STUDENT SEMINAR TOPICS**

1. Special functions and their applications.
2. Analytic Functions and their applications.
3. Residence Theorem and their applications.

**ASSIGNMENT**

**7.1.12 QUESTION BANK**

**UNIT - I**

**UNIT - II**

**1.** State and prove generating function for $J_n(x)$. **(DEC 13)**

**2.** a) Prove that $(2n+1)\,P_n(x) = P^1_{n+1}(x) - P^1_{n-1}(x)$.

b) Prove that $\int_{-1}^{1} xP_n(x)P_{n-1}(x)\,dx = \frac{2n}{4n^2-1}$. **(DEC 13)**

**3.** Prove that $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x) + \left(1 - \frac{24}{x^2}\right)J_0(x)$ (May-13)

4. Prove that $x^2 \; J''_n(x) = (n^2-n-x^2)\,J_n(X) - x\,J_{n+1}(x)$.

**(Dec 12)**

5. Show that $\frac{n}{x} J_n(x) + J_n(x) = J_{n-1}(x)$ **(Dec 12)**

6 Express $J_2(x)$ interms of $J_0(x)$ & $J_1(x)$. **(Dec 11)**

7. S.T. $J_{5/2}(x) = \sqrt{\frac{2}{®x}}\left\{\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right\}$ **(Dec 11)**

*Creation is a drug I can't do without.*

\- Cecil B. DeMille

8. i. Express the following interms of legendre polynomials $4x^3-2x^2-3x+8$.

ii. Evaluate $\int_{-1}^{1} x^2 U_3(x)dx.$ **(Dec11)**

9. i P. T. Jn(X) = 0 has no repeated roots except at x - 0 **(Nov 10)**

ii. P. T. $\frac{d}{dx}\{x J_1(x)\} = x J_0(x)$

10. i. S.T. $J_{3/2}(x) = \sqrt{\frac{2}{®x}}\{1/x \sin x - \cos x\}$ **(Nov 10)**

ii. S.T. $\int_0^1 x^3 (1-\sqrt{x})^5 dx = 2$ (8,6)

11. P.T. $xJ_n^1(x) = nJn(x) - (X) - xJn + 1(X)$ **(Nov 10)**

12. Show that $J_4$ (x) = $\left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x)$ **(Feb 08)**

13. ii. Prove that $\frac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}(x)$

iii. When n is an integer, show that $\mathrm{J}_{-n}(\mathrm{x}) = (-1)^n \mathrm{J}_n(\mathrm{x})$ **(Feb 08)**

14. Show that when ‘n’ is a positive integer, $J_n$ *(x)* is the coefficient of $z^n$ in the expansion of $\exp\left\{\frac{x}{2}\left(z - \frac{1}{z}\right)\right\}$.

**(Feb 08, Nov 07, May 01)**

15. Prove that J0 (x) = $1 - \frac{x^2}{2^2} + \frac{x^4}{2^2.4^2} - \frac{x^6}{2^2.4^2.6^2} + \ldots\ldots\ldots.$ **(Nov 07)**

16. Prove that $(2n+1)(1-x^2)P_n^1(x) = n(n+1)(P_{n+1}(x) - P_{n-1}(x))$ **(Feb, Nov 07)**

17. Prove that $\frac{d}{dx}(xJ_n J_{n+1}) = x(J_n^{\ 2} - J_{n+1}^{\ 2})$ **(Feb 07)**

18. Prove that $J\frac{3}{2}(x) = \sqrt{\frac{2}{®x}}\left(\frac{\sin x}{x} - \cos x\right)$ **(Nov 10)**

19. Prove that $\frac{n}{x} J_n(x) + J_n^{\ 1}(x) = J_{n-1}(x)$ **(Feb 07)**

20. Prove that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right)\ldots\ldots\ldots\ldots\Gamma\left(\frac{n-1}{n}\right) = \frac{(2\Pi)^{\frac{n-1}{2}}}{n^{1/2}}$. **(May 11,Feb 07, May 06)**

21. Prove that $J_0^2 + 2(J_1^2 + J_2^2 + \ldots\ldots\ldots\ldots) = 1$ **(Nov 06, 03)**

*The merit of originality is not novelty; it is sincerity.*

- Thomas Carlyle

22. Prove that $J_2 - J_0 = 2J_0^{11}$ **(Nov, May 06, Apr 05)**

23. Show that $4J_n^{11}(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$. **(May 11, May 06)**

24. When n is a positive integer show that $J_n(x) = \frac{1}{n}\int_0^{®} \cos(n^{-} - x\sin^{-})d^{-}$ . **(May 06)**

25. Write $J_{\frac{5}{2}}(x)$ in finite form. **(May 06)**

26 a) Prove that $\int_{-1}^{1} x^2 P_{n+1} P_{n-1} dx = \frac{2n(n+1)}{(2n+1)(2n-1)(2n+3)}$

b) Prove that $(1-x^2)T_n^1(x) = nT_{n-1}(x) - nxT_n(x)$ (May-13)

27. a) Express $x^3$-2$x^2$+ x+2 in terms of Legendre polynomial.

b) Show that Pn (x) = $\frac{1}{2^n Xn!}\frac{d^n}{dx^n}(x^2-1)^n$. **(Dec12)**

28. a) Prove that $\frac{1+z}{z\sqrt{1-2xz+z^2}} - \frac{1}{z} = \sum_{n=0}^{\tilde{}} [P_n(x) + P_{n+1}(x)]z^n$

b) Prove that (2n+1) x$P_n$(x) = (n+1) $P_{n+1}$(x)+n$P_{n-1}$(x) **(Dec12)**

29. i. P.T. $P_n^1(-1) = (-1)^{n-1}\frac{n(n+1)}{2}$

ii. Write $T_2$(x) + $T_1$(x) +$T_0$(x) as a polynomial. **(Dec11)**

5. i. P.T. $(1-x^2)\,p_n^1(x) = (n+1)\{xp_n(x) - P_{n+1}(x)\}$

ii. Express the following interms of legendre polynomials 1+x-$x^2$. **(Dec11)**

30. i. S.T. x3 = $\frac{2}{5}$ P3(x) + $\frac{3}{5}$ P1(x).

ii. Express f(x) = 2x+10x3 interms of legendre polynomials. **(Dec11)**

31. i. Express $x^3$+3$x^2$+4x+3 interms of Legendre polynomial.

ii. Evaluate $\int_0^1 x(1-x^2)^{-\frac{1}{2}}U_4(x)dx$. **(May 11)**

32. If f(x) = $\begin{matrix} 0 & ; & 1 < x & 0 \\ x & ; & 0 < x < 1 \end{matrix}$ then **(Nov 10)**

*It is wise to learn; it is God-like to create.*
- John Saxe

S.T. $f(x) = \frac{1}{4} P_0(x) + \frac{1}{2} P_1(x) + \frac{5}{16} P_2(x) \quad \frac{3}{32} P_4(x) + - -$

33\. i. P.T. $\int_{-1}^{1} \left(P_n^1\right)^2 dx = n(n+1)$

ii. $P_n^1 - P_{n-2}^1 = (2n-1)P_{n-1}$ **(Nov 10)**

34\. i. Evaluate $\int_{-1}^{1} x^4 (1-x^2)^{-1/2} T_2(x)dx$ **(Nov 10)**

ii. Prove that $\int_{-1}^{1} x^2 P_{n-1}(x) P_{n+1}(x)dx = 0$

35\. i. Prove that, for all $x^7 = \frac{116}{429} P_7(x) + \frac{8}{9} P_5(x) + \frac{14}{33} P_3(x) + \frac{1}{3} P_1(x)$

ii. Show that $\int_{-1}^{1} x^k P_n(x)dx = 0$ for K = 0, 1,2, n-1 **(Nov 10)**

36\. Show that $\int_{-1}^{1} (1-x^2)\,(P'_n)^2\, dx = \frac{2n(n+1)}{(2n+1)}$ **(Feb 08)**

37\. Establish the formula $P'_{n+1}(x) - P'_{n-1}(x) = (2n+1)\,P_n(x)$ **(Feb 08)**

38\. i. Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0; m \neq n \\ \frac{2}{2n+1}; m = n \end{cases}$ **(Feb 08, Nov 07, Jan 03)**

ii. $\int_{-1}^{1} x^n P_n(x)dx = \frac{2^{n+1}.(n!)^2}{(2n+1)!}$ **(Nov 07)**

39\. Prove that $\int_{-1}^{1} xP_n(x)P_{n-1}(x) = \frac{2n}{(4n^2-1)}$ **(Nov 07)**

40\. Prove that $(1-x^2)\,p_n^1(x) = (n+1)[xP_n(x) - P_{n+1}(x)]$ **(Feb, Nov 07)**

41\. Express $x^3 + 2x^2 - x - 3$ in terms of Legendre Polynomials **(Feb 07)**

42\. Prove that $\int_{-1}^{1} (x^2-1)P_{n+1}P_n^1 dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$ **(Feb 07)**

43\. Prove that $\frac{1}{\sqrt{1-2tx+t^2}} = P_0(x) + P_1(x)t + P_2(x)t^2 + .....$ **(May 11, Nov ,May 06, Jan 03)**

44\. Prove that $P_n(0) = 0$ for n odd and $P_n(0) = \frac{(-1)^{\frac{n}{2}} n!}{2^n \left(\frac{n}{2}!\right)^2}$ if n is even. **(Nov, May 06, Apr 05)**

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*Man was made at the end of the week's work when God was tired.*

\- Mark Twain

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45. Show that $x^4 = \frac{8}{35}P_4(x) + \frac{4}{7}P_2(x) + \frac{1}{5}P_0(x)$ **(Nov 06)**

46. Show that $x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$. **(May 06)**

47. Using Rodrigue's formula prove that $\int_{-1}^{1} x^m P_n(x)dx = 0$ if m < n. **(Nov 10)**

**UNIT-III**

**1.** Show that the function $f(x) = \begin{cases} \frac{x^3 y(y-ix)}{x^6+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not differentiable at the **orgin.** **(Dec 13)**

**2** a) Evaluate $\int_0 (x^2 + iy)\, dz$ along the path y = x and y = x².

b) Use Cauchy's Integral formula to evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$, where 'C' is the circle $|z-i| = 2$. [15] **(Dec 13)**

3 a) Find the analytic function whose real part is $e^{2x}(xCos2y - ySin2y)$

b) Show that the function defined by f(Z) = $\frac{x^3(1+i) - y^3(1-i)}{(x^2+y^2)}$ at $Z \neq 0$ ,and f(0) = 0 is continuous and satistfies C.R equations at the origin, but $f^1(0)$ does not exist. (May 13)

4. a) Prove that U = $x^2 y^2$; V= $\frac{y}{x^2+y^2}$ are harmonic functions of (x,Y) but are not harmonic conjugates.

b) Find an analytic function whose real part is $e^{-x}$ (xsin y - cos y) **(Dec12)**

5. a) find the analytical function whose real part is r²Cos2¯ + rSin¯ .

b) If f(Z) is an analytic function of z, Prove that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2$ **(Dec12)**

6. i. If W = ¤ + $i$ · represents the complex potential for an electric eld & · = $x^2 - y^2 + \frac{x}{x^2+y^2}$, determine the function ¤ .

ii. If f(z) = u+iv is an analytic function of z & u-y = $e^x$ (cosy-siny) nd f(z)interms of z. **(Dec11)**

7. i. Find whether the function u = log |z |² is harmonic. If so, nd the analyticfunction whose real part is u.

ii. Separate the real and imaginary parts of $i^{\log}$ (1+i).

*Ideas are the root of creation.*

\- Ernest Dimnet

8. i. If f = u + iv is analytic in a domain D and uv is constant in D, then provethat f(z) is constant.

ii. Find the general and primial values of log(1+i $\sqrt{3}$ ).

**(Dec11)**

9. i. P.T. The function f(z) dened by f(z) = $\begin{cases} \dfrac{x^3(1+i)-y^3(1-i)}{x2+y2}; & (z \neq 0) \\ 0 ; & (z=0) \end{cases}$ is continuousand the C-R equa tions are satised at the origin, yet f1(0) does not exists. **(Dec11)**

10. Find the analytic function f(z) = u(r, θ)+ i v(r, θ) such that u(r; θ) = $r^2 \cos 2\theta - r\cos\theta + 2$.

ii. S.T. The function u= 1/2 log ($x^2 + y^2$) is harmonic & find its conjugate. **(May 11)**

11. i. S.T. the real & imaginary parts of the function w = log z satisfy the C-R equations when z is not zero.

ii. S.T. f(z) = z+2$\overline{z}$ is not analytic anywhere in the complex plane. **(May 11)**

12. S.T. both the real & imaginary parts of an analytic function are harmonic. **(May 11)**

13. i. S.T. $\tan^{-1} z = \frac{1}{2}\log\frac{i+z}{i-z}$

ii. Find the roots of sinz = cosh4 **(Nov 10)**

14. i. Prove that **tan -¹** z = $\left[\log\frac{(i+z)}{(i-z)}\right]$ **(Nov 10)**

ii. Find the analytic function whose real part is $r^{-4}\cos 4$

15. Find the analytic function whose real part is $\frac{y}{x^2+y^2}$ **(Nov 10)**

16. i. S.T. the real & imaginary parts of the function w = log z satisfy the C - R equations when z is not zero.

ii. S.T. f(z) = z + 2 $\overline{z}$ is not analytic anywhere in the complex plane. **(Nov 10)**

17. i. Find the analytic function whose real part $\mu = e^{2x}\left[x\cos 2y - y\sin 2y\right]$

ii. Find whether $f(z) = \sin x \sin y - i\cos x\cos y$ is analytic or not

18. i. Find the modulus and argument of $i^{\log(1+i)}$

ii. Show that an analytic function of constant module is constant. **(May 11)**

19. i. Prove that the function f(z) = $\sqrt{|xy|}$ is not analytic at the origin even though the C - R equations are satisfied thereat.

ii. Find the analytic function whose real part is y / ($x^2 + y^2$). **(Nov 09, 08)**

20. i. Derive Cauchy Riemann equations in polar coordinates. **(Nov 09,Feb 08, May 05, Jan 03)**

ii. Prove that the function f(z) = conjugate of z is not analytic at any point.

*Great effort is required to arrest decay and restore vigor. One must exercise proper deliberation, plan carefully before making a move, and be alert in guarding against relapse following a renaissance.* - I Ching

21. i. Show that w = $z^n$ (n, a positive integer) is analytic and find its derivative. **(Nov09,May 05)**
    ii. Prove that $\mu = e^{x^2-y^2}$ is a harmonic function and find its harmonic conjugate.

22. Find whether f(z) = $\dfrac{x-yi}{x^2+y^2}$ is analytic or not. **(Nov09,May 02)**

23. Find the general and principal values of (i) $\log_e(1+\sqrt{3}\,i)$ (ii) $\log_e(-1)$ **(Nov 09,Feb 08, Nov 04)**

24. Separate the real and imaginary parts of tan hz. **(Nov 09,May 06)**

25. Separate the real and imaginary parts of sin hz. **(Nov 09,May 06)**

26. i. Find the real and imaginary parts of Cot z.
    ii. Prove that $e^{\bar{z}}$ is analytic
    iii. Find $e^z$ and $|e^z|$ if $z=4\pi(2+i)$ **(Nov 09)**

27. Solve tan h+2=0 **(Nov 09)**

28. If tan(x+iy)=A+iB show that $A^2+B^2-2B\cot h^2Y+1=0$ **(Nov 11)**

29. i. Show that the function defined by f(z) = $\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ at z $\neq 0$ and f(0)=0.is continuous and satisfies CR equations at the origin but $f^1(0)$ does not exist **(Nov 08, 06, Feb 07, May 01)**
    ii. Find the analytic function whose real part is y +$e^x$ cos y.

30. i. Show that $f(z)=\dfrac{xy^2(x+iy)}{x^2+y^2},(x,y)\neq(0,0)$ and 0 at (x,y)=(0,0)is not analytic at z = 0 although CR equations are satisfied at the origin **(Nov 08, Feb 07)**
    ii. If w = $\phi+i\psi$ represents the complex potential for an electric field and $\psi = 3x^3y-y^3$ find $\phi$.

31. The necessary and sufficient conditions for the function f(z) = u(x, y) + i v( x, y) to be analytic in the region R, are **(Nov 08)**
    i. $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial v}{\partial x}, \dfrac{\partial v}{\partial y}$ are continuous functions of x and y in R.
    ii. $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}, \quad \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

32. i. Prove that
    a. $i^i = e^{-(4n+1)\pi/2}$
    b. $\log i^i = -(2n+\frac{1}{2})\pi$
    ii. If tan(x + iy) = A + iB, Show that $A^2 + B^2 + 2A\cot 2x = 1$. **(Nov 08)**

*I am opposed to millionaires, but it would be dangerous to offer me the position.*
- Mark Twain

33. i. Separate into real and imaginary parts of cosh ( x + iy ) . **(Nov 08)**
ii. Find all the roots of the equation
a. sin z = cosh 4
b. sin z = i.

34. i. Find the real part of the principal value of $i^{\log(1+i)}$ **(Nov 08)**
ii. Separate into real and imaginary parts of sec (x + i y).

35. i. Separate into real and imaginary parts of coth z.

ii. If tan 1og (x + iy) = a + ib where $a^2 + b^2 \ {}^{4}_{4}\ 1$ prove that tan log $(x^2 + y^2) = \dfrac{2a}{1-a^2-b^2}$ **(Nov 08, May 05)**

36. If sin (α+iβ) = x + iy then prove that $\dfrac{x^2}{\cosh^2\beta} + \dfrac{y^2}{\sinh^2\beta}$ = 1 and $\dfrac{x^2}{\sin^2\alpha} - \dfrac{y^2}{\cos^2\alpha}$=1. **(Feb 08)**

37. Determine the analytic function f(z) = u+iv given that $3u + 2v = y^2 - x^2 + 16x$. **(Feb 08)**

38. Test for analyticity at the origin for f(z) = $\dfrac{x^3y(y-ix)}{x^6+y^2}$ for $z \neq 0$. **(Feb 08, May 06)**

= 0
for z = 0.

39. Find all values of z which satisfy (i) $e^z = 1 + i$ (ii) sinz = 2. **(Feb 08, May 06, Nov 03)**

40. If w = f(z) is an analytic function, then prove that the family of curves defined by u(x,y) = constant cuts orthogonally the family of curves v(x,y) = constant. **(Feb 08,May 05)**

41. Show that $f(x,y) = x^3y - xy^3 + xy + x + y$ can be the imaginary part of an analytic function of z = x + iy. **(Feb 08, May 05)**

42. Prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)|\text{Real } f(z)|^2 = 2\,|f'(z)|^2$ where w = f(z) is analytic. **(Feb 08, May 05)**

43. Find f(z) = u + iv given that u + v = $\dfrac{\sin 2x}{\cosh 2y - \cos 2x}$ **(Feb 08, Jan 03)**

44. Derive Cauchy Riemann equations in cartesian co-ordinates. **(Feb 07, Nov 06, May, Jan 03)**

45. If $(x+iy)^{\frac{1}{3}} = a+ib$ then prove that $4(a^2-b^2) = \dfrac{x}{a}+\dfrac{y}{b}$ **(Feb 07)**

46 a) Evaluate $\int_0^{1+i}(x^2-iy)dz$ along i)The straight line y = x ii) Along $y = x^2$

b) Evaluate $\int_c \dfrac{(z+4)dz}{(z^2+2z+5)}$ where C is $|z+1-i|$=2 (May 13)

"*When students are frustrated or get involved in put-down behavior, Random Acts of Kindness works beautifully.*"
- Author:Kathy, Teacher, CA

47.a) Evaluate $\int_{(1,1)}^{(2,8)} (x^2 + ixy)dz$ along x = t and y = $x^3$.

b) Using Cauchy's integral formula, evaluate $\int_c \frac{zdz}{(z-1)(z+3)}$ where C: |z| = 1.5. **(Dec12)**

48. a) Evaluate $\oint_c \frac{z-1}{(z+1)^2(z-2)}dz$ where C:|z-i|=2

b) Evaluate $\int_c \frac{z^4}{(z+1)(z-1)^2}dz$ Where 'C' is the ellipse $9x^2+4y^2=36$. **(Dec12)**

49. i. Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(1=x^2)(x^2+4)}dx$

ii. Evaluate by Residue theorem $\int_c \frac{\sin z}{z^6}dz$ where C: |z| = 2 **(Dec11)**

50. i. Evaluate $\int_{(0,0)}^{(1,1)} ((x-y^2)dx + 2xydy)$ along $x_0$ curve y = x

ii. Evaluate $\int_{z=0}^{z=1+i} \{x^2 + 2xy + i(y^2 - x)\}$ dz along y = $x^2$ **(Dec 11)**

51. i. Evaluate $\int_c \frac{z^3+z^2+2z-1}{(z-1)^3}$ dz where C: |z| = 3

ii. Evaluate $\int_c \frac{z^4}{(z+1)(z-i)^2}dz$ where 'C' is the ellipse $9x^2+4y^2=36$ **(Dec 11)**

52. Evaluate $\int_c \frac{z-3}{z^2+2z+5}$ where C is

i. $|z| = 1$
ii. $|z+1-i| = 2$
iii. $|z+1+i| = 2$

ii. Evaluate $\int_c \frac{5z^2 - 3z + 2}{(z-1)^3} dz$ where c is any simple closed curve enclosing z = 1. **(Dec11)**

53. i. Evaluate $\int_c \frac{3z^2 + 7z + 1}{z+1} dz$ where C: $|z + i| = 1$

ii. Evaluate $\int_c \frac{z^2 - z + 17z + 1}{z-1} dz$ where C: $|z| = \frac{1}{2}$ taken in antiiclockwise sense **(Dec 11)**

54. i. Evaluate ò$_c$ $(z^2 + 3z + 2)dz$ where C is the arc of the cycloid x=a(q + sin), y= a(1-cosq) between the points (0,0) & (ap, 2a)

ii. Evaluate ò$_c$ $(z^2 + 3z)dz$ along the straight line from (2,0) to (2,2) and then from (2,2) to (0,2). **(May 11)**

55. The only singularities of a single valued function f (z) are poles of order 1 and 2 at z = -1 and z = 2 with residues at these poles i and 2 respectively. If f (0) = 7/4 , f(1) = 5/4 , determine the function f (z).**(May 11)**

56. Let `C' denotes the boundary of the square whose sides lie along the lenes x = $\pm 2$, y = $\pm 2$ where `C' is described in the positive sense evaluate the following integrals **(May 11)**

i. $\int_C \frac{\tan(z/2)}{(z-x_0)^2} dx \; (|x_0| < 2)$

ii. $\int_C \frac{\cosh z}{z^4} dz$

57. From the integral $\int_0 \frac{dz}{z+4}$ S.T $\int_0^\pi \frac{1+4\cos\theta}{17+8\cos\theta} - 0$ where C: $|z| = 1$.

ii. If C is a closed curve described in + ve sense and $f(z_0) = \int_c \frac{z^4 + z}{(z - z_0)^4} dz$ show that $f(z_0) = 8piz_0$ is where $z_0$ is a point inside `C' and $f(z_0) = 0$ if $z_0$ lies outside `C'. **(May 11)**

58. i. Evaluate $\int_{1-i}^{z+i} (2x + iy + 1) dz$ along the straight line joining (1,-i) & (2,i)

ii. If C is the boundary of the square with vertices at the points z = 0, z = 1, z=1+i and z = i show that

$\int_c (3z + 1) dz = 0)$ **(Nov 10)**

59. i. Evaluate $\int_c (x + y)dx + x^2 y$ dy along y=3x between (0,0) and (3,a)

ii. Evaluate $\int_c e^z dz$ where C: $|z| = 1$ **(Nov 10)**

*Clothes make the man. Naked people have little or no influence on society.*
- Mark Twain

60. i. Evaluate $\int_0^{3+i} z^2 dz$ along the line x=2y

ii. Use Cauchy's integral formula to evalute $\oint \frac{e^z}{(z+2)(z+1)^2}dz$ where C is the circule $|z| = 3$

61. i. a. Prove that $\int_C \frac{dz}{z-a} = 2®i$ where C is $|z-a| = r$ **(Nov 09, 08, 06)**

b. Evaluate $\oint_c (z-a)^n dz;\text{for } n=-1, n \neq -1.$ n is integer.

ii. State and prove Cauchy's integral theorem.

62. Evaluate $\int_{(0,0)}^{(1,1)} (3x^2 + 4xy + ix^2)dz$ along $y = x^2$. **(Nov 09, 06, 04,Feb 08)**

63. i. Evaluate $\int_0^{1+i}$ $(x^2 + iy)$ dz along the path y = x and $y = x^2$. **(Nov 09, 08)**

ii. Evaluate, using Cauchy's integral formula $\int_c \frac{e^{2z}}{(z-1)(z-2)}dz$, where c is the circle |z| = 3.

64. Evaluate $\oint_c \frac{dz}{(z-a)^n}$ if z = a is point inside a simple closed curve C and n is integer. **(Nov 09, 06, May 00)**

65. Evaluate using Cauchy's theorem $\int_c \frac{z^3 e^{-z} dz}{(z-1)^3}$ where c is | z - 1 | = 1/2 using Cauchy's integral formula.

**(Nov 09,May 06)**

66. i. Evaluate $\int_0^{3+i}$ $z^2$dz, along

a. the line y = x/3

b. the parabola $x = 3y^2$

ii. Evaluate $\int_c \frac{z^3 - 2z + 1}{(z-i)^2}$ where C is | z | ⬌ 2 by using Cauchy's Integral Formula. **(Nov 08, Jan 03)**

67. i Evaluate $\int_0^{1+i} (x^2 - iy)$ along $y = x^2$ and y = x **(Nov 10)**

ii. Use Cauchy's integral formula to evaluate $\oint_c \frac{\sin^2 z}{\left(z-\frac{\pi}{6}\right)^3}dz$ where c is the circle |z| = 1.

68. i. Evaluate $\int_C \frac{(z^3 - \sin 3z)\ dz}{(z-\frac{\pi}{2})^3}$ with C: | z | = 2 using Cauchy's integral formula.

*Ethical axioms are found and tested not very differently from the axioms of science. Truth is what stands the test of experience.*
- Albert Einstein

ii. Evaluate $\int_C \frac{dz}{e^z(z-1)^3}$ where C: | z | = 2 Using Caucy's integral theorem. **(Feb 08)**

69. Prove Cauchy's theorem. **(Feb 08, 07)**

70. Evaluate $\int_c \frac{\log z \ dz}{(z-1)^3}$ where c : $|z-1| = \left(\frac{1}{2}\right)$, using Cauchy's Integral Formula. **(Nov10)**

71. Evaluate using Cauchy's integral formula $\int_c \frac{(z+1)dz}{z^2+2z+4}$ where c : | z + 1 + i| = 2. **(Nov 07, 06, May 06)**

72. Evaluate using Cauchy's integral formula $\int_c \bar{z}dz$ from z = 0 to 4 + 2i along the curve C given by

i. $z = t^2 + it$

ii. Along the line z = 0 to z = 2: and then from z =2 to z = 4 + 2i. **(Nov 07, May 06)**

73. i. Find f(z) and f(3) if f(a)= $\int_C \frac{(2z^2-z-2)dz}{(z-a)}$ where C is the circle |z| = 2.5 using Cauchy's integral formula.

ii. Evaluate $\int_C \log z dz$ where C is the circle |z| = 1 using Cauchy's integral formula. **(Nov 07)**

74. i. Evaluate $\int_{-2+i}^{2+i} z^3 dz$ using Cauchy's integral formula along y = x.

ii. $\int (x+y)dx + ix^2$ along $y=x^2$ from (0,0) to (3,9).

iii. Evaluate $\int_{-1+i}^{2+i}$ $(x^2 - y^2 + i\,xy)\,dz$ using Cauchy's integral formula along $y=x^2$. **(Nov 07)**

75. i. Evaluate using Cauchy's integral formula $\int_c \frac{(z+1)dz}{z^2+2z+4}$ where C : | z + 1 + i| = 2.

ii. Evaluate $\int_c \bar{z}dz$ from z = 0 to 4 + 2i along the curve C given by

a. $z = t^2 + it$

b. Along the line z = 0 to z = 2: and then from z =2 to z = 4 + 2i. **(Nov 07)**

76. If F(a) = $\int_C \frac{3z^2+7z+1}{z-a}dz$ where C is | z | = 2 find F(1), F(3) , $F^{11}(1-i)$ using Cauchy's Integral formula **(Feb 07)**

77. Evaluate $\int_C \frac{z^3 - \sin z}{(z-\frac{®}{2})^3}dz$ with C:| z | =2 using Cauchy's integral formula **(Feb 07)**

*In the beginning the Universe was created. This has made a lot of people very angry and been widely regarded as a bad move.*
- Douglas Adams

78. Evaluate $\int_C \frac{dz}{e^z (z-1)^3}$ where C:| z | =2using Cauchy's integral formula **(Feb 07)**

79. Evaluate $\int_C \frac{dz}{z^2 e^z}$ where C: | z | =1 **(Feb 07)**

80. Evaluate using Cauchy's integral formula $\int_0^{1+i} z^2 dz$ along $y = x^2$ **(May 11, Feb 07)**

81. Evaluate $\int_C \frac{\log z}{(z-1)^3} dz$ where C: $|z-1| = \frac{1}{2}$ **(Feb 07)**

**UNIT-IV**

1. a) Find the Taylor's series expansion of $e^z$ about z = 3.
   b) Expand $f(z) = \frac{z^2-4}{z^2+5z+4}$ in the region $1 < |z| < 4$. **(Dec 13)**

2. a) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$, where 'C' is the circle $|z| = \frac{3}{2}$ using Residue theorem.
   b) Evaluate by contour Integration $\int_0^\infty \frac{dx}{1+x^2}$. [15] **(Dec 13)**

3 a) Expand $\frac{z}{(z+1)(z+2)}$ About Z = 2.

b) Expand $\frac{4z+3}{(z-3)(z+2)}$ in the annular region between $|z|$ =2 and $|z|$ =3 (May 13)

4. a) Find a Taylor Series expansion of a function f(z)= $\frac{1}{(z-1)(z-3)}$ about the point z=4. Obtain the region of convergence. **(Dec12)**

b) Determine the poles and residue at each pole of the function f(z) = $\frac{z^2}{(z-1)^2 (z-2)}$.

4. a) Find the Tailor's expansion for the function f(z)= $\frac{1}{(1+z)^2}$ with centre at –i.

b) Find the Laurent's series of $\frac{1}{z^2-4z+3}$ for 1< z <3. **(Dec12)**

5. State and prove Taylor's Theorem of complex function f (z). **(Dec11)**

6. i. Represent the function f (z) = $\frac{1}{z(z+2)^3(z+1)^2}$ in Laurent series with in $\frac{5}{4} \leq |z| \leq \frac{7}{4}$

ii. Dene for a complex function (z)

i. Isolated Singularity

ii. Removable Singularity

iii. Essential singularity

**(Dec11)**

7. Expand $\frac{7z-2}{(z+1)z(z-2)}$ about the point z = -1 in the region 1 < |z + 1| < 3 as Laurent'sseries.

**(Dec11)**

8. i. For the function f(z) = $\frac{2z^3+1}{z^2+z}$ find

(a) Find the Taylsor's series expansion of about z =3.

(b) Explain f(z)= Cos z in Taylor's series about z = $\frac{®}{2}$ **(Dec11)**

9. Expand $f(z) = \frac{z+3}{z(z^2-z-2)}$ in powers of z.

i. With in the unit circle about the origin

ii. With in the annular region between the concentric circles about the origin having radii 1 and 2 respec tively.

iii. The exterior to the circle of radius 2. **(May 11)**

10. Expand the function f (z) = $\frac{4z+4}{z(z-3)(z+2)}$ in powers of z, when **(May 11)**

i. $|z| \leq 1$

ii. $1 \leq |z| \leq 2$

iii. $|z| > 2$

11. Expand f (z) = $\frac{z+3}{z(z^2-z-2)}$ in powers of z.

i. With the in the unit circle about the origin **(Nov 10)**

ii. With in the annular region between the concentric circle about the origin having radii 1 and 2 respectively

iii. The exterior to the circle of radius 2

12. For the function $\int(z) = \frac{2z^2}{z^2+z}$ find

i. A tailors expansion valid in the neighborhood of the point 'i'.

ii. A Laurent's series valid within the annulus of which centre is origin **(Nov 10)**

13. i. Represent the function f (z) = $\frac{1}{z(z+2)^3(z+1)^2}$ in Laurent series iwth in $\frac{5}{4} \leq |z| \leq \frac{7}{4}$

ii. Define for a complex function (z)

*Never hold discussions with the monkey when the organ grinder is in the room.*

- Sir Winston Churchill

a.Isolated Singularity
b. Removable Singularity
c. Essential Singularity **(Nov 10)**

14. Obtain the Laurent's series expansion of $f(z) = \frac{e^z}{z(1-3z)}$ About z = 1 **(Nov 09)**

15. i. Expand f(z) = $\frac{1+2z}{z^2+z^3}$ in a series of positive and negative powers of z. **(Nov 09, 08)**
ii. Expand $e^z$ as Taylor's series about z = 1.

16. i Find Laurent's series for f(z) = $\frac{1}{z^2(1-z)}$ and find the region of convergence. **(Nov 09, 08, May 06)**

ii. Expand $\frac{1}{z^2-3z+2}$ in the region ( i ) 0 < | z -1| < 1 and (ii) 1< | z | < 2 **(Nov 09, 08, Feb 07)**

17. i. State and derive Laurent's series for an analytic function f(z). **(Nov 09,Feb 08, Nov 07, May 06)**

ii. Obtain Taylor's in the expansion of $\frac{e^z}{z(z+1)}$ about z=2

18. Expand f(z) = $\frac{1}{z^2-z-6}$ about (i ) z = -1 (ii) z = 1 **(Nov 09, 06)**

19. Obtain Taylor series to represent the function $\frac{z^2-1}{(z+2)(z+3)}$ in the region | z | < 2. **(May 11,06, Nov 09)**

20. i. Expand log (1 - z) when |z| < 1 **(Nov 08)**
ii. Determine the poles of the function
a. $\frac{z}{\cos z}$
b. cot z.

21. i. Expand f(z) = z-1/z+1 in Taylor's series about the point z = 0 and z = 1. **(Nov 08)**

ii. Determine the poles of the function $\frac{1-e^{2z}}{z^4}$

22. Expand the Laurent series of $\frac{z^2-1}{(z+2)(z+3)}$ for 1 < | z | < 3. **(Feb 08)**

23. Evaluate $f(z) = \frac{z}{z^2+1}$ where C is | z | = 3/2. **(Feb 08)**

24 Expand log z by Taylor series about z =1 **(Feb 08, 07, Nov 07)**

25. Expand $\frac{1}{(z^2+1)(z^2+2)}$ in positive and negative powers of z if 1< | z | < $\sqrt{2}$ **(Feb 08, 07, Nov 07)**

26. Obtain all the Laurent series of the function $\frac{7z-2}{(z+1)z(z-2)}$ about z = -2 **(Feb 08, 07, Nov 07)**

27. Find the Taylor series of $\frac{z}{z+2}$ about z = 1, also find the region of convergence. **(Feb 08, Apr 05)**

28. Find the Laurent expansion of $\frac{1}{z^2-4z+3}$, for 1 < | z | < 3. **(Nov 07, May 06)**

29. Expand f(z) = $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region (a) 1 < | z | < 4 (b) | z | <1 **(Feb, Nov 07)**

31. Find the poles and the residues at each pole of $f(z)=\frac{z}{z^2+1}$. **(Nov 07)**

32. Find the Laurent series expansion of the function **(Nov 07, 06, 03)**

i. $\frac{z^2-1}{z^2+5z+6}$ about z = 0 in the region 2 < | z | < 3.

ii. $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region 3 < | z + 2 | < 5. **(May 11)**

33. State and prove Taylor’s theorem. **(Feb 07, Nov, May 06, Nov 04, May 02)**

34. Show that when $|z+1|<1, z^{-2}=1+\sum_{n=1}^{\infty}(n+1)(z+1)^n$. **((May 11, Nov 06, 04, 03)**

35 a) Evaluate $\int_0^{2®}\frac{d¯}{5-3\cos¯}$ by contour integration in complex plane. (May 13)

36. a) Evaluate $\int_0^{®}\frac{1}{3+2\cos¯}d¯$ by Contour integration in the complex plane.

b) Evaluate $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+1)(x^2+4)}dx$ by method of complex variables. **(Dec12)**

37. a) Evaluate $\int_{-\infty}^{\infty}\frac{x^2-x+2}{x^4+10x2+9}dx$ using residue theorem.

b) Evaluate by Residue theorem $\int_C\frac{z-1}{(z+1)^2(z-2)}dz$ where C: |z-I|=2. **(Dec12)**

38. Find the bilinear transformation which maps z1 = 1, 2i; $z_2$ = 2 + i; $z_3$ = 2 + 3iin to the points $w_1$ = 2 + 2i;$w_2$ = 1 + 3i;$w_3$ = 4 respectively. Find the fixed and critical points. **(Dec11)**

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*As been my experience that folks who have no vices have very few virtues.*

\- Abraham Lincoln

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39. i. Evaluate $\int_0^\infty \frac{1}{1+x^6}dx$

ii. Evaluate by Residue theorem $\int_c \frac{4z^2-4z+1}{(z-2)(z^2+4)}dz$ C: |z| = 1. **(Dec11)**

40. i. By the method of contour integration prove that $\int_0^\infty \frac{\cos mx}{a^2+x^2}$ dx = $\frac{®}{2}e^{-ma}$

ii. Evaluate by Residue theorem $\int_c \frac{z^2}{(z-1)^2(z-2)}$ dx were |z|=3. **(Dec11)**

41. i. Evaluate $\int_0^\infty \frac{\sin x}{x^2+4x+5}dx$

ii. Evaluate by Residue theorem $\int_c \frac{z-1}{(z+1)^2(z-2)}dz$ where C:|z - i| = 2 **(Dec11)**

42. i. Evaluate $\int_0^\infty \frac{\log x}{1+x^2}\,dx$

ii. Find the Residues of f (z) = $\frac{1}{z(e^z-1)}$. **(May 11)**

43. i. Using complex variable techniques evaluate $\int_0^{2\pi} \frac{\sin^2\theta\, d\theta}{5-4\cos\theta}$

ii. The only singularities of a single valued function f(z) are poles of order 2 and 1 at z =1 & z = 2 with residues of these poles as 1 and 3 respectively. If f (0) = 3/2 , f (-1) = 1, determine the function. **(May 11)**.

i. Find the residue of $\frac{1}{(z-Sinz)}$ at z = 0

ii. Evaluate $\int \frac{(z-3)dz}{(z^2+2z+5)}$ Where C is $|z+1-i|=2$ **(Nov 10)**

44. i. Find the image of the triangle with vertices at i, 1+i, 1-i in the z - plane, under the transformation

$e^{\frac{5®i}{3}}.(z-2+4i)$

*It is not a pleasant condition, but certainty is absurd.*

\- Voltaire

ii. Find the image of the infinite strip, $0<y<\frac{1}{2}$ under the mapping function $w=\frac{1}{z}$ **(Nov 10)**

45. i. Evaluate $\int_c \frac{z^4+z^2+2z-1}{(z-1)^3}dz$ where C: $|z| = 3$

ii. Evaluate $\int_c \frac{z^4}{(z+1)(z-i)^2}dz$ where ‘C’ is the Ellipse $9x^2+4y^2=36$ **(Nov 10)**

46. i. Using the method of contour integration prove that $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+1)^2}dx$

ii. Determine the poles and the residues at each pole of the function f(z) = Cot Z **(Nov 10)**

47. i Find the Residues of f (z) = $\frac{z^2}{z(z+2)^3}$ $at$ Z = -2

ii. Find the Residues of f (z) = $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $|z|=1$ **(Nov 10)**

48. i. Define conformal Transformation. Show that a bilinear transformation is conformal.

ii. Show that circles are invariant under linear transformation w = az + c. **(Nov 10)**

49. i. State and prove Cauchy’s Residue theorem. **(Nov 09,08)**

ii. Calculate the residue at z = 0 of $f(z)=\frac{1+e^z}{z\cos z+\sin z}$,

50. i. Determine the poles of the function f(z) = z2 / (z+1)2(z+2) and the residues at each pole. **(Nov 09,08)**

ii. Evaluate $\oint_c \frac{dz}{(z^2+4)^2}$ where c = |z - i| = 2

51. Find the poles of the function $f(z)=\frac{1}{(z+1)(z+3)}$ and residues at these poles.

52. Show by the method of residues, $\int_0^\pi \frac{d\theta}{a+b\cos\theta}=\frac{\pi}{\sqrt{a^2-b^2}}$ (a > b > 0). **(Nov 09, 08)**

53. Evaluate $\int_0^{2®}\frac{\sin^2{}^{-}d^{-}}{a+b\cos^{-}}$ **(Nov 09, Feb 07, May 2000)**

*Doubt is not a pleasant condition, but certainty is an absurd one.*

- Voltaire

54. i. Find the residues of $f(z)=\dfrac{z^3}{(z-1)^4(z-2)(z-3)}$ at z = 1

ii. Show that $\displaystyle\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}=\frac{2\pi}{\sqrt{3}}$ **(Nov 10)**

55. Evaluate $\displaystyle\int_0^{2\pi}\frac{d\theta}{(5-3\sin\theta)^2}$ using residue theorem. **(Nov 08)**

56. Show by the method of contour integration that $\displaystyle\int_0^{\infty}\frac{Cosmx}{(a^2+x^2)}dx=\frac{®}{4a^3}(1+ma)e^{-ma}$ (a > 0, b > 0). **(Feb 08)**

57. Evaluate by contour integration $\displaystyle\int_0^{\infty}\frac{dx}{(1+x^2)}$. **(Feb 08)**

58. i. Evaluate by residue theorem $\displaystyle\int_0^{2®}\frac{d¯}{(2+\cos¯)}$

ii. Use the method of contour integration to evaluate $\displaystyle\int_{-\infty}^{\infty}\frac{x^2dx}{(x^2+a^2)^3}$. **(Feb 08)**

59. i. Show that $\displaystyle\int_0^{2®}\frac{d¯}{a+bSin¯}=\int_0^{2®}\frac{d¯}{a+bCos¯}=\frac{2®}{\sqrt{a^2-b^2}}$, a > b > 0 using residue theorem.

ii. Evaluate $\displaystyle\int_0^{\infty}\frac{dx}{(x^2+a^2)^2}$ **(Feb 08, 07, Nov, May 04)**

60. Find the poles and residues at each pole of f (z ) = $\dfrac{\sin^2 z}{(z-\frac{®}{6})^2}$ **(Feb 08, Nov 06)**

61. Find the poles and residues at each pole of $\dfrac{ze^z}{(z-1)^3}$ **(Feb 08, Nov, May 06)**

62. Determine the poles of the function $\dfrac{z+1}{z^2(z-2)}$ and the corresponding residues **(Feb 08, May 06)**

63. Evaluate $\displaystyle\oint_c\frac{dz}{Sinhz}$, where c is the circle | z | = 4 using residue theorem. **(Feb 08, May 05)**

64. Show that $\displaystyle\int_0^{®}\frac{Cos2¯}{1-2aCos¯+a^2}=\frac{®a^2}{\sqrt{1-a^2}}$, ($a^2 < 1$) using residue theorem. **(Feb 08, May 05)**

*One can survive everything, nowadays, except death, and live down everything except a good reputation.*

\- Oscar Wilde

65. Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ **(Feb 08, Nov 07, May 04, 01 Jan 03)**

66. Determine the poles of the function $\frac{(2z+1)^2}{(4z^3+z)}$ and the corresponding residues **(Nov 07, May 05)**

67. Evaluate $\int_C \frac{(\sin ® z^2 + \cos z^2)dz}{(z-1)^2(z-2)}$ where c is the circle | z | = 3 using residue theorem. **(Nov 07, May 05)**

68. Evaluate $\int_0^{2®} \frac{d¯}{(a+b\cos¯)^2}$ , a > b > 0, using residue theorem. **(Nov 07, May 05)**

69. Using the complex variable technique evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^4+1}$ **(Nov 07, May 2000)**

70. Evaluate $\int_C \frac{ze^2 dz}{(z^2+9)}$ where c is | z | = 5 by residue theorem. **(Nov 07)**

71. Evaluate $\int_0^{2®} \frac{Cos2¯}{5+4Cos¯} d¯$ using residue theorem. **(Nov 07)**

72. Evaluate $\int_0^{\infty} \frac{dx}{(x^2+1)^3}$ using residue theorem. **(Nov 07)**

73. Evaluate $\int_C \frac{ze^z dz}{(z^2+9)}$ where C is | z | = 5 by residue theorem. **(Nov 07)**

74. Evaluate $\int_0^{\sim} \frac{dx}{(x^2+1)^3}$ **(Nov 07, May 05)**

75. Show that $\int_0^{®} \frac{ad¯}{a^2+\sin^2¯} = \frac{®}{\sqrt{1+a^2}}$ , (a>0) using residue theorem. **(Nov 07, 04)**

76. Determine the poles and the corresponding residues of $f(z) = \frac{z+1}{z^2(z-2)}$ **(Feb 07)**

77. Evaluate $\oint_C \frac{dz}{\sinh z}$ , C: | z | =4 using residue theorem **(Feb 07)**

*One should always play fairly when one has the winning cards.*
- Oscar Wilde

78. Evaluate by Countour integration $\int_0^\infty \frac{dx}{1+x^2}$ **(Feb 07)**

79. Find the poles and the residues at each pole of $f(z)=\frac{1-e^z}{z^4}$ **(Feb 07)**

80. Evaluate $\int_C \frac{z-3}{z^2+2z+5}dz$ where C is the circle ( i ) | z |=1 ( ii ) | z+1-i | =2 using residue theorem **(Feb 07)**

81. Evaluate by residue theorem $\int_0^\infty \frac{dx}{x^6+1}$ using residue theorem **(Feb 07)**

82. Evaluate $\int_0^{2®} \frac{d¯}{(5-3\cos¯)^2}$ using residue theorem **(Feb 07)**

83. Evaluate $\int_0^\infty \frac{\sin mx}{x}dx$ by residue theorem **(Feb 07)**

**UNIT-V**

1. a) Show that the function $w=\frac{1}{z}$ transforms the straight line x=c in the z-plane in to a circle in the w-plane.
   b) Under the transformation $w=\frac{z-i}{1-iz}$ find the image of the circle
   (i) $|w|=1$ (ii) $|z|=1$ in the w-plane. [15] (Dec13)

2 a) Show that the transformation $W=z^2$ maps the circle $|z-1|=1$ into the cardiod $° = 2(1+\cos¯)$ where $W = °e^{i¤}$
  b) Find the bilinear mapping which maps the points z=-1,-i,-1 to i,0,-i (May13)

3. a) Find the image of the strip $\frac{-®}{2}<x<\frac{®}{2}$; 1< y<2 under the mapping w(z) = sin(z). Draw the rough sketch of the regions in Z-plan e and W-plane.
  b) Find the bilinear transformation that maps the points $z_1$ = -I, $_z2$ =0, $z_3$ =I in to the points $w_1$= -1, $w_2$= I, $w_3$ =1I respectively. **(Dec 12)**

4. a) find the image of the circle |z-2i|=2 in the complex plane under the mapping w= $\frac{1}{2}$.
  b) Show that the transformation w= $\frac{i(1-z)}{1+z)}$ transforms the circle |z| =1 in to the real axis in the w-plane and the interior of the circle into upper half of the w-plane. **(Dec 12)**

*Search others for their virtues, thyself for thy vices.*
- Benjamin Franklin

5. Find the bilinear transformation which maps $z_1 = 1 - 2i$, $z_2 = 2 + i$, $z_3 = 2 + 3i$ in to thepoints $w_1 = 2 + 2i$, $w_2 = 1 + 3i$, $w_3 = 4$ repectively. Find the fixed and critical points. **(Dec 11)**

6. i. Show that the transformation w = cos z maps the half of the z-plane to theright of the imaginary axis intotheentire w-plane.
   ii. Show that the transformation maps the family of line parallel to y-axis toellipse. **(Dec11)**

7. i. Find the points at which w = cosh z is not conformal.
   ii. Find the image of the strip bounded by x = 0 and x = $\pi/4$ under the transfor- mation w = cos z. **(May 11)**

8. Find the bilinear transformation which maps $z_1 = 1$; $z_2 = i$; $z_3 = -1$ in to the points $w_1=i$; $w_2 = 0$;$w_3 = -i$ respectively. Find the fixed and critical points of this transformation and find the image of $|z| < 1$. **(May 11)**

9. i. Find the image of the triangle with vertices at i,1+i,1-i in the z-plane, under the transformation $e^{\frac{5\pi i}{3}}(z-2+4i)$.
   ii. Find the image of the infinite strip, $0 < y < 1/2$ under the mapping function w = 1/z . **(May 11)**

10. i. Show that the transformation $w = \frac{3-z}{z-2}$ transforms the circle $\left|z - \frac{5}{2}\right| = \frac{1}{2}$ in the z-plane in to the imaginary axis in the w-plane.
    ii. For the mapping w = 1/z, find the image of the family of circles $x^2 + y^2 = ax$,where a is real. **(May 11)**

11. i. Find the map of the circle |z| =C under the transformation w = z - 2 + 4i
    ii. Show that both the transformations $w = \frac{z-i}{z+i}$ and $w = \frac{i-z}{i+z}$ transforms $|w| \le 1$ into upper half planeI (z) >0. **(Nov 10)**

12. i. Find the image of |z| = 2 under the transformation w = 3z.
    ii. Show that the transformation $w = z^2$ maps the circle |z-1| = 1cardiod r = 2(1+Cox‾ **(Nov 10)**

13. i. State that the transformation $W=e^z$ transform the region between the real axis and the line paralel to the real axis at $y=\pi$ into upper half of the W-plane.
    ii. Find the bilinear transformation which maps Z=-1, i, 1 into the point W=-i, 0, i. **(Nov 09 )**

14. Define conformal mapping ? State and prove the sufficient condition for W = f(z) to be conformal at the point $z_0$. **(Nov 09 07, Jan 03)**

15. Find and plot the image of triangular region with vertices at (0,0), (1,0) (0,1) under the transformation w= (1-i) z + 3. **(Nov 09,06, May 05, Nov 04)**

16. i. Find the image of the region in the z-plane between the lines y = 0 and y = $\frac{®}{2}$ under the transformation $w = e^z$. **(Nov 09,May 06)**
    ii. Find the bilinear transformation which maps the points (-1, 0, 1) into the points. (0, i, 3i)

*A fanatic is one who can't change his mind and won't change the subject.*
*- Sir Winston Churchill*

17. Under the transformation w = $\frac{1}{z}$ find the image of the circle | z-2i | = 2. **(Nov 09,May 06)**

18. Show that the transformation w = z+1/z , converts that the radial lines θ= constant in the z-plane in to a family of confocal hyperbolar in the w-plane. **(Nov 08)**

19. i. Find and plot the image of the regions **(Nov 08)**
 a. x > 1
 b. y > 0
 c. 0 < y < 1/2 under the transformation w = 1/z
 ii. Prove that every bilinear transformation maps the totality of circle and straight lines in the z - plane on to the totality of circles and straight lines in the w-plane.

20. i. Show that horizantal lines in z - plane are mapped to ellipser in w - plane under the transformation w = sin z.
 ii. Define Bilineer transformation. Determine the Bilinear transformation which maps z = 0, -i, 2i into w = 5i, oo, -i/3. **(May 11,Nov 08)**

21. Find the image of the infinite strip 0 < y < 1/2 under the transformation w = $\frac{1}{z}$ .
**(Nov 08, 07, Feb 08, 07, May 05)**

22. Show that the image of the hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$ is $r^2 = \cos 2$
**(Nov10)**

23. i. Show that the transformation w= i(1–z) / (i–z), maps the interior of the circle |z|=1 in to the upper half of the w-plane, the upper semi circle into positive half of real axis and lower semi circle into negative half of the real axis.
 ii. By the transformation w = $z^2$ show that the circle | z–a | = c (a and c are real) in the z plane correspond to the limacons in the w-plane. **(Feb 08)**

24. Find the bilinear transformation which maps the points (-1, 0, 1) into the points (0,i,3i).
**(Feb 08, 07, Nov 07, May 05)**

25. Find the image of the domain in the z-plane to the left of the line $x = -3$ under the transformation $w = z^2$
**(Feb 08, Nov 06)**

26. Find the bilinear transformation which maps the points z = 2, 1, 0 into w = 1, 0, 1 respectively.
**(Feb 08,Nov 06)**

27. Show that the transformation $w = \frac{2z+3}{z-4}$ changes the circle $x^2 + y^2 - 4x = 0$ into the line 4u+3=0
**(Feb, Nov 07)**

28. Find the bilinear transformation which maps the pints (0, 1, ∝) into the points (-1, -2, -i).
**(Nov 07, 06, May 06, 2000)**

29. Let f(z) be analytic function of z in a domain D of the z-plane and let $f'$ (z) $\frac{4}{4}$ 0 in D. Then show that w = f(z) is a conformal mapping at all points of D. **(Nov 07, 03)**

30. Find the bilinear transformation which maps the points (–i, o, i) into the point (–l, i, l) respectively. **(Nov 07)**

*Although prepared for martyrdom, I preferred that it be postponed.*
- Sir Winston Churchill

30. Under the transformation $w = \frac{z-i}{1-iz}$, find the image of the circle | z | = 1 in w-plane. **(Feb 07)**

31. Show that the mapping $w = z + \frac{1}{z}$ maps the circle | z | =c into the ellipse $u = (c + \frac{1}{c})\cos^{-}$, $v = (c - \frac{1}{c})\sin^{-}$. Also dicuss the case when c = 1 in detail. **(Feb 07)**

*Broadly speaking, the short words are the best, and the old words best of all.To profit from good advice requires more wisdom than to give it.* - John Churton Collins